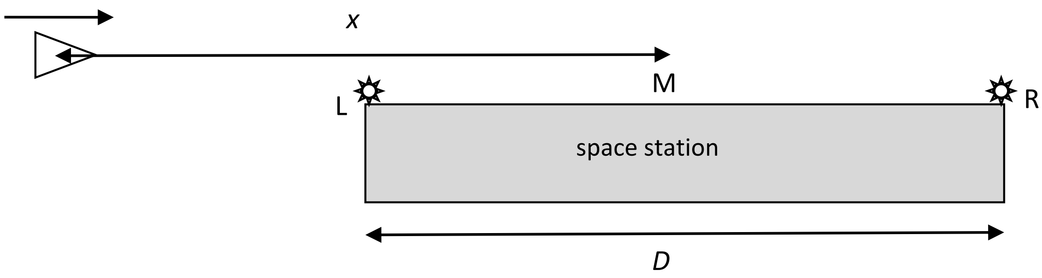
Teacher notes
Topic A

Is seeing believing or the beauty of spacetime diagrams

This problem discusses the difference between the actual time of an event and the time an observer sees photons from that event.

A rocket approaches a space station of proper length *D*. Two lights are turned on at the right (R) and left (L) of the space station. The lights are turned on at the same time according to space station clocks.



At the instant the lights are turned on according to the space station the rocket is at a distance *x* from the mid-point M of the space station according to the space station.

(a) Explain which light turns on first according to the rocket.

(b) Using a spacetime diagram or otherwise, show that the light from the two lamps reaches the rocket at the *same time* when $x = \dfrac{vD}{2c}$.

Answers

(a) We can answer this in many ways:

Method 1: using a Lorentz transformation

$$\Delta t' = t'_{\mathrm{R}} - t'_{\mathrm{L}}$$

$$\Delta t' = \gamma(\Delta t - \frac{v}{c^2}\Delta x) = \gamma(0 - \frac{v}{c^2}L) = -\gamma\frac{v}{c^2}L$$

$t'_{\mathrm{R}} - t'_{\mathrm{L}} < 0$ so R lights before L.

Method 2: using the concept of relativistic simultaneity

An observer at rest at M receives light from R and F at the same time.

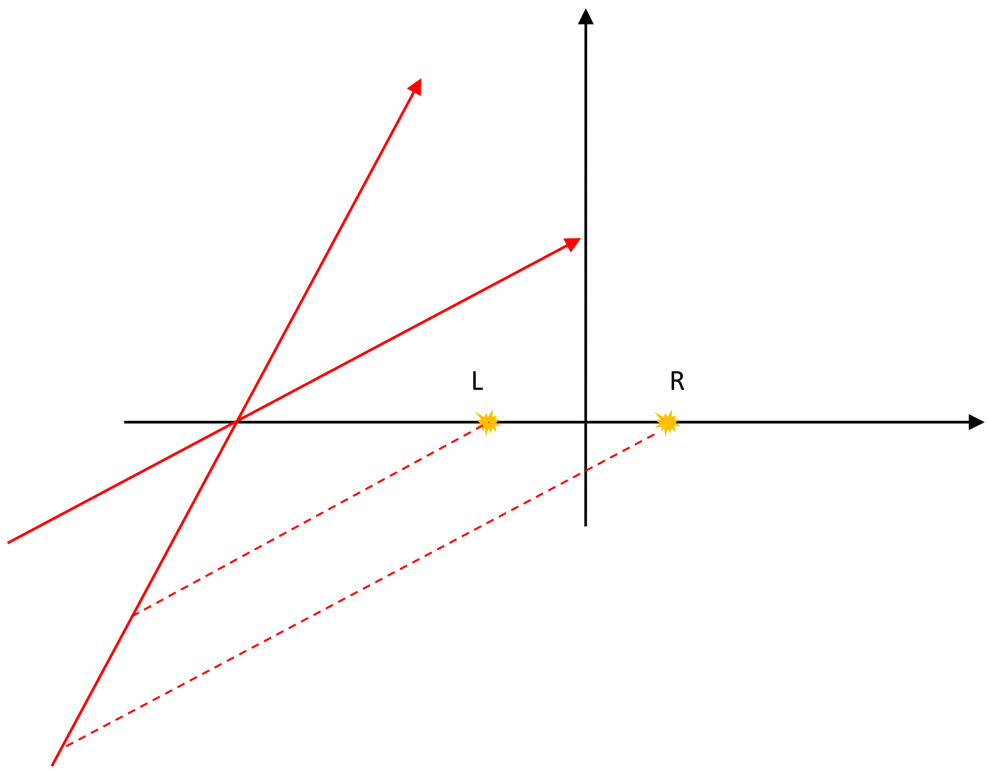
I.e. the arrivals of light from L and R are simultaneous for this observer and happen at the same point in space. Therefore, they are simultaneous for all other observers as well.

The rocket observer is free to consider themselves at rest and sees M coming towards them.

So, M moves away from the light from R and towards the light from L.

For the rocket observer to measure that the light arrives at the same time implies that the light from R was emitted first since light moves at the same speed and has a longer distance to cover.

Method 3: using a spacetime diagram, the easiest method of all!



The dotted lines (parallel to the rocket space axis) intersecting the rocket time axis give the time of the lights turning on in the rocket frame. Clearly, R lights before L.

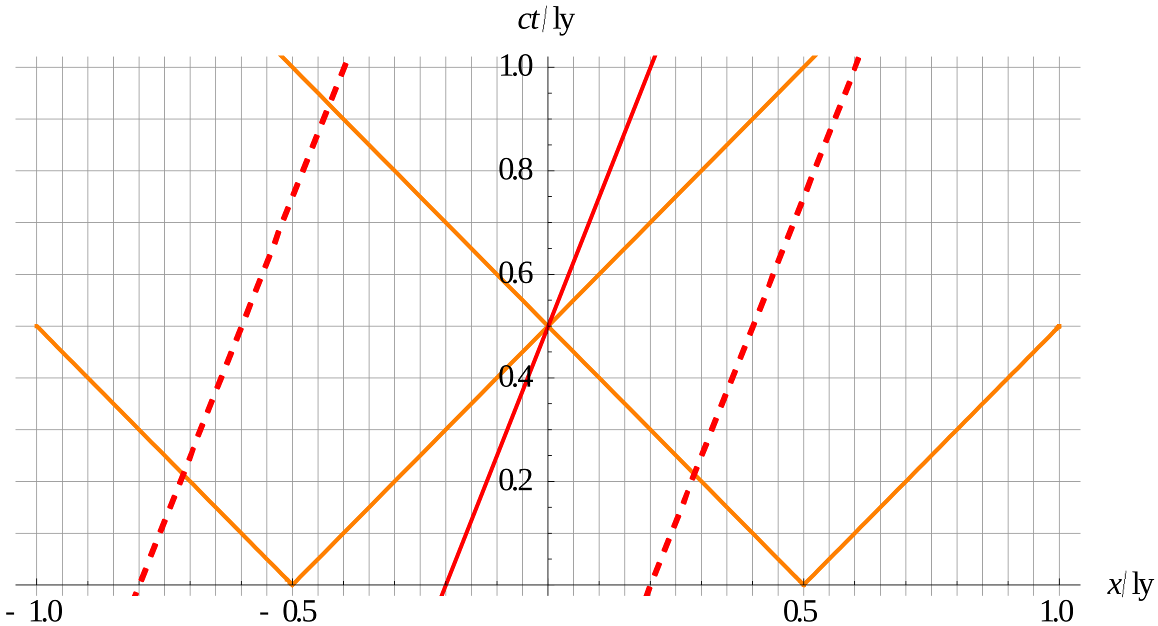
(b) Using a spacetime diagram here is very instructive. The orange lines are photon worldlines leaving the lamps which have been placed at $x = \pm 0.5$ ly . The red lines are worldlines of the rocket for three different positions of the rocket at t = 0.

The leftmost red line clearly receives light from L before light from R.

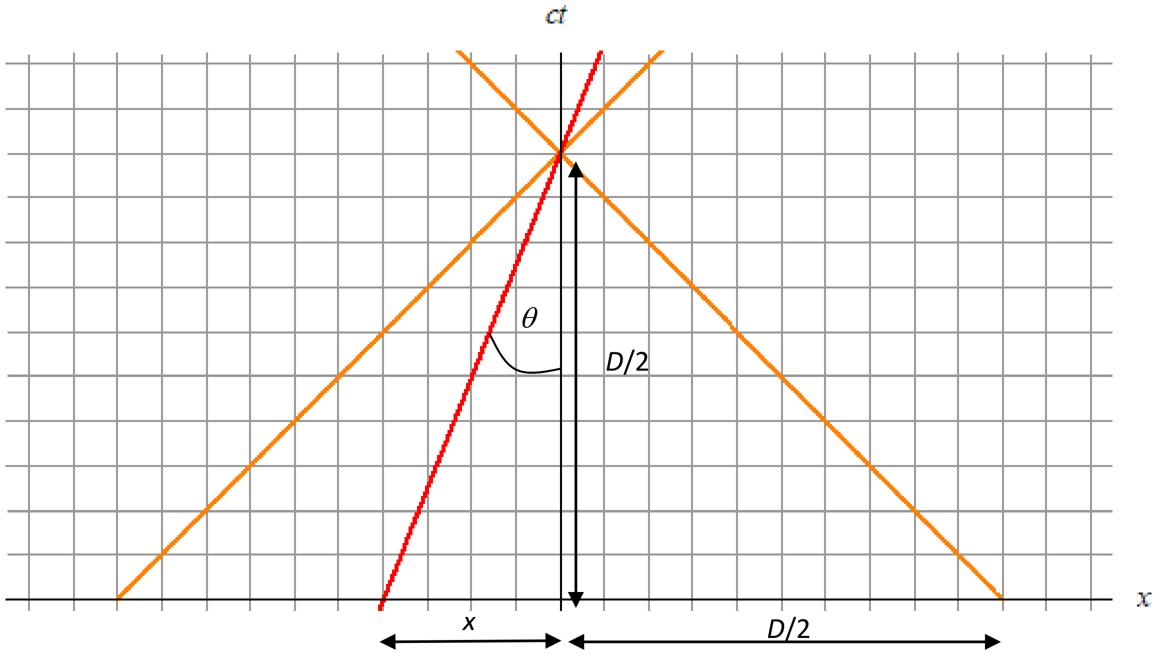
The rightmost red line receives light from R before light from L.

The middle solid red line receives light at the same time.

Clearly, the photon worldlines from L and R intersect along the space station time axis and so the rocket worldline must pass through that point if the rocket is to get light from the 2 lamps at the same time. The distance *x* in this case is then 0.2 ly.



How do we treat the general case?



$\tan\theta = \frac{v}{c}$ and $\tan\theta = \frac{x}{\frac{D}{2}} = \frac{2x}{D}$.

Hence

$$\frac{2x}{D} = \frac{v}{c} \quad \Rightarrow \quad x = \frac{vD}{2c}$$

The special case we drew the spacetime diagram for corresponded to *v* = 0.4c and *D* = 1 ly. Hence $x = \frac{vD}{2c} = \frac{0.4c \times 1}{2c} = 0.2\text{ ly}$ just as the diagram said.